Hong Kong Mathematics Olympiad (1984 – 85)

Sample Event (Individual)

香港数学竞赛 (1984 – 85)

决赛项目 – 样本 (个人)

(i) The sum of two numbers is 40, and their product is 20. If the sum of their reciprocals is a, find a.

$a =$

某两数之和为 40，其积为 20。 若该两数倒数之和为 a，求 a。

(ii) If b cm^2 is the total surface area of a cube of side $(a + 1)$ cm, find b.

$b =$

若一边长 $(a + 1)$ cm 之正方体之总表面积为 b cm^2，求 b。

(iii) One ball is taken at random from a bag containing $b - 4$ white balls and $b + 46$ red balls. If $\frac{c}{6}$ is the probability that the ball is white, find c.

$c =$

一袋内有 $b - 4$ 个白球，$b + 46$ 个红球。若随意于袋内取一球，而该球为白色之概率为$\frac{c}{6}$，求 c。

(iv) The length of a side of an equilateral triangle is c cm. If its area is $d\sqrt{3}$ cm^2, find d.

$d =$

若一边长 c cm 之正三角形之面积为 $d\sqrt{3}$ cm^2，求 d。

Hong Kong Mathematics Olympiad (1984 – 85)

Event 1 (Individual)

香港数学竞赛 (1984 – 85)

决赛项目 1 (个人)

(i) Find a if $a = \log_5 \frac{(125)(625)}{25}$.

$a =$

若 $a = \log_5 \frac{(125)(625)}{25}$，求 a。

(ii) If $\left(r + \frac{1}{r}\right)^2 = a - 2$ and $r^3 + \frac{1}{r^3} = b$, find b .

$b =$

若 $\left(r + \frac{1}{r}\right)^2 = a - 2$ 且 $r^3 + \frac{1}{r^3} = b$，求 b。

(iii) If one root of the equation $x^3 + cx + 10 = b$ is 2 , find c .

$c =$

若 2 为方程 $x^3 + cx + 10 = b$ 之一根，求 c。

(iv) Find d if $9^{d+2} = (6489 + c) + 9^d$.

$d =$

若 $9^{d+2} = (6489 + c) + 9^d$，求 d。

Hong Kong Mathematics Olympiad (1984 – 85)

Event 2 (Individual)

香港数学竞赛 (1984 – 85)

决赛项目 2 (个人)

(i) Find a in the following sequence:

$1, 8, 27, 64, a, 216, \ldots\ldots$

$a =$

在以下数列中，求 a:

$1, 8, 27, 64, a, 216, \ldots\ldots$

(ii) In Figure 1, AC = CD and $\angle CAB - \angle ABC = (a - 95)^\circ$. If $\angle BAD = b^\circ$, find b.

$b =$

在图一中，AC = CD，$\angle CAB - \angle ABC = (a - 95)^\circ$。若 $\angle BAD = b^\circ$，求 b。

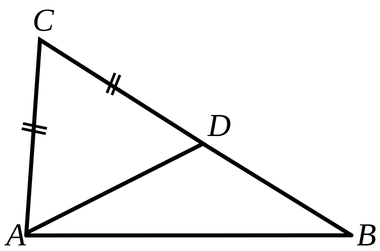

Figure 1
图一

(iii) A line passes through the points $(-1, 1)$ and $(3, b - 6)$. If the y-intercept of the line is c, find c.

$c =$

一直线过 $(-1, 1)$ 及 $(3, b - 6)$。若其 y 截距为 c，求 c。

(iv) In Figure 2, $AB = c + 17$, $BC = 100$, $CD = 80$. If $EF = d$, find d .

在图二中， $AB = c + 17$ ，$BC = 100$，$CD = 80$。若 $EF = d$，求 d。

$d =$

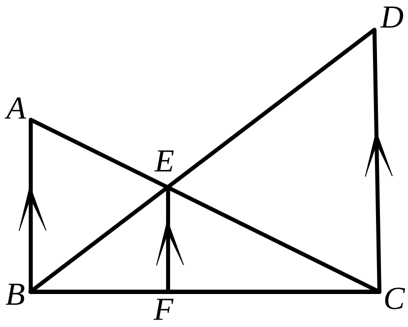

Figure 2
图二

Hong Kong Mathematics Olympiad (1984 – 85)

Event 3 (Individual)

香港数学竞赛 (1984 – 85)

决赛项目 3 (个人)

(i) The acute angle formed by the hands of a clock at 2:15 is $\left(18\frac{1}{2}+a\right)^{\circ}$. Find a.

$a =$

在二时十五分，时钟两针所构成之锐角为 $\left(18\frac{1}{2}+a\right)^{\circ}$，求 a。

(ii) If the sum of the coefficients in the expansion of $(x+y)^a$ is b, find b.

$b =$

若 $(x+y)^a$ 的展开式之系数总和是 b，求 b。

(iii) If $f(x)=x-2$, $F(x, y)=y^2+x$ and $c=F(3, f(b))$, find c.

$c =$

若 $f(x)=x-2$，$F(x, y)=y^2+x$，且 $c=F(3, f(b))$，求 c。

(iv) x, y are real numbers. If $x+y=c-195$ and d is the maximum value of xy, find d.

$d =$

x，y 为实数。若 $x+y=c-195$ 及 d 为 xy 之最大值，求 d。

Hong Kong Mathematics Olympiad (1984 – 85)

Event 4 (Individual)

香港数学竞赛 (1984 – 85)

决赛项目 4 (个人)

(i) If the lines $x+2y+3=0$ and $4x-ay+5=0$ are perpendicular to each other, find a.

$a =$

若两线 $x+2y+3=0$ 及 $4x-ay+5=0$ 互相垂直，求 a。

(ii) In Figure 1, $ABCD$ is a trapezium with AB parallel to DC and $\angle ABC=\angle DCB=90°$. If $AB=a$, $BC=CD=8$ and $AD=b$, find b.

$b =$

在图一中， $ABCD$ 为一梯形， AB 与 DC 平行且 $\angle ABC=\angle DCB=90°$。若 $AB=a$，$BC=CD=8$ 及 $AD=b$，求 b。

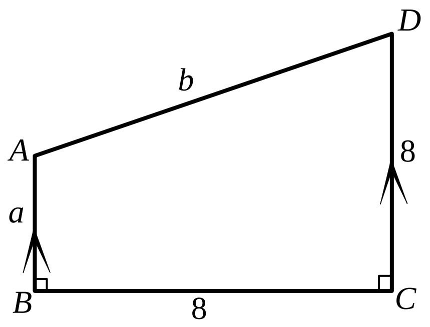

Figure 1
图一

(iii) In Figure 2, $BD=\frac{b}{2}$, $DE=4$, $EC=3$. If the area of ΔAEC is 24 and the area of ΔABC is c, find c.

$c =$

在图二中， $BD=\frac{b}{2}$，DE = 4，EC = 3。 若 ΔAEC 之面积为 24 及 ΔABC 之面积为 c，求 c。

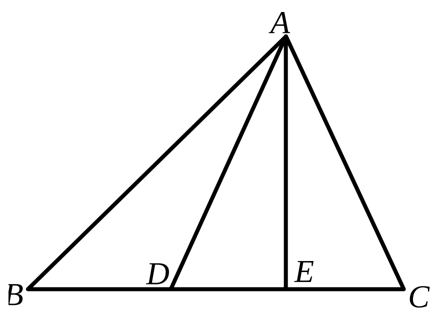

Figure 2
图二

(iv) If $3x^3-2x^2+dx-c$ is divisible by $x-1$, find d.

$d =$

若 $3x^3-2x^2+dx-c$ 可被 $x-1$ 整除，求 d。

Hong Kong Mathematics Olympiad (1984 – 85)

Event 5 (Individual)

香港数学竞赛 (1984 – 85)

决赛项目 5 (个人)

(i) If $1+2+3+4+\cdots+t=36$, find t.

若 $1+2+3+4+\cdots+t=36$，求 t。

$t =$

(ii) If $\sin u^\circ = \dfrac{2}{\sqrt{t}}$ and $90 < u < 180$, find u.

若 $\sin u^\circ = \dfrac{2}{\sqrt{t}}$ 且 $90 < u < 180$，求 u。

$u =$

(iii) In Figure 1, $\angle ABC = 30^\circ$ and $AC = (u-90)$ cm. If the radius of the circumcircle of ΔABC is v cm, find v.

在图一中，$\angle ABC = 30^\circ$，且 AC $= (u-90)$ cm。若 ΔABC 之外接圆半径为 v cm，求 v。

$v =$

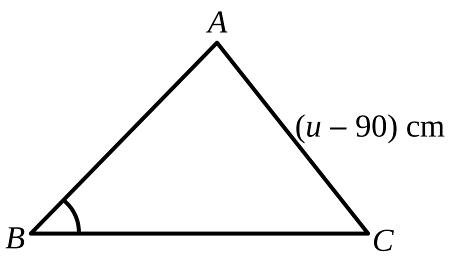

Figure 1
图一

(iv) In Figure 2, ΔPAB is formed by the 3 tangents of the circle with centre O. If $\angle APB = (v-5)^\circ$ and $\angle AOB = w^\circ$, find w.

在图二中，ΔPAB 由切于圆 O 之三切线形成，若 $\angle APB = (v-5)^\circ$，且 $\angle AOB = w^\circ$，求 w。

$w =$

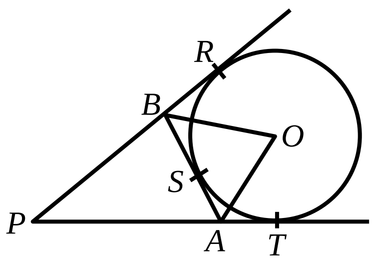

Figure 2
图二